

Strong-coupling scale and frame-dependence of the initial conditions for chaotic inflation in models with modified (coupling to) gravity

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Abstract

A classical evolution in chaotic inflationary models starts at high energy densities with semi-classical initial conditions presumably consistent with universal quantum nature of all the fundamental forces. That is each quantum contributes the same amount to the energy density. We point out the upper limit on this amount inherent in this approach, so that all the quanta are inside the weak-coupling domain. We discuss this issue in realistic models with modified gravity, R^2 - and Higgs-inflations, emphasizing the specific change of the initial conditions with metric frame, while all the quanta still contribute equal parts. The analysis can be performed straightforwardly in any model with modified gravity ($F(R)$ -gravity, scalars with non-minimal couplings to gravity, etc).

1. The idea of early time inflation—an epoch when the Universe expands almost exponentially—has been put forward [1, 2, 3, 4] to solve the problems of initial conditions in the Hot Big Bang theory. Namely, the Universe becomes as we know its—flat, homogeneous, isotropic and populated with adiabatic scalar perturbations—at the inflation preceding the hot stages. However, a particular model of inflation can be recognized as one truly solving the initial condition problems only if realization of the exponential expansion occurs naturally or likely: may be a special, but reasonably large part of the initial phase space of the dynamical variables leads to the sufficiently fast expansion.

Evidently, the issue of naturalness includes a task of outlining the available part of the model phase space to start the evolution from. The other tasks are estimating the relative probability of beginning from a particular point in the phase space and evaluating the subsequent dynamics. The higher the probability *to start the evolution ending in inflation*, the more natural the given inflationary model.

The naturalness issue can be properly addressed in models of chaotic inflation [5, 6]. There the dynamics starts from initial conditions attributed to the semi-classical description of quantum processes in a spatial cell of the Planck-scale size. The assumption is that all the ingredients—like curvature for gravity and potential, kinetic, and gradient energies for scalar fields—give roughly the same amount to the total energy density. The amount is naturally expected to be of order one in Planck units. In each cell these quantities fluctuate around the Planck value. If accidentally the potential term becomes somewhat bigger than others, the cell expands, the energy density decreases, but the contributions of kinetic and gradient terms do it much faster, leaving the region dominated by the scalar potential [7, 8, 9]. Then the energy density dominates, which makes the expansion fast, almost exponential.

2. This scheme works perfectly for the power-law potentials like original $\lambda\phi^4$ [5], but seemingly fails for inflation models with flat, plateau-like potential favored by the present cosmological data. The value of the scalar flat potential at inflation is constrained from above by the negative searches for imprints of the relic gravitational waves—tensor perturbations arisen at the inflationary stage. The allowed value is much below the Planck scale, hence the beginning with all the terms being of order one in Planck units is simply impossible in these models. If the gradient and the kinetic terms dominate, the Universe expands but moderately, insufficiently for initiating an inflationary stage and hence for solving the Hot Big Bang problems. Only the regions, where the kinetic and gradient terms take enormously small values, so that the potential dominates, can evolve into inflationary regime. However, such a fluctuation is extremely rare, and hence the chaotic inflation with a flat potential is very unlikely.

In particular, one of the most developed models of this type— R^2 -inflation [1] and Higgs-inflation [10]—were accused of suffering from this unlikeness [11]. Recently it has been realized [12] that, quite the contrary, within the paradigm of chaotic inflation what must be treated as very unnatural is the situation, when the kinetic and gradient terms much exceed the potential one. The chaotic inflation paradigm implies that all the terms must be of the same order, and hence about the value V_0 of the flat scalar potential. Recall, this value is fixed from the cosmological observations.

The key point of Ref. [12] is that the hierarchy between the contributions of different ingredients to energy density remains the same independently of the metric frame. Since both of the models under discussion exhibit rather non-trivial coupling to gravity, the recognition of the proper ingredients (degrees of freedom of the quantum system) is subtle in the so-called Einstein frame, where the gravity itself is Einsteinian (General Relativity) and the scalar potential is flat. The situation becomes much clearer in the Jordan frames, where the models have been originally formulated. There, as in any other frames, the properly defined ingredients give the same amount to the energy density. However, this amount, related by the Weyl transformation to the fixed by observations value V_0 of the scalar potential, happens to be not confined to be (right) below the Planck scale. Moreover, if in the Einstein frame one takes the gradient, kinetic or gravity terms to be of the Planck scale, then upon the Weyl transformation, in the Jordan frame the energy density grossly exceeds the strong coupling gravity scale. This peculiarity deserves a little investigation, which bring us to the main subject of this note, that is frame-dependence of the chaotic initial conditions.

3. The starting point here is the motivation for limiting from above the term contributions to the total energy density. First, whichever cell is taken, its spatial scale Δx defines the critical scale of energy fluctuations ΔE in it through the quantum uncertainty relation $\Delta E \Delta x \geq 2\pi$. Second, one constrains all the quantities to be inside the weak coupling domain, otherwise we simply cannot properly define the quantity itself. For the case of General Relativity and scalar field (inflaton) minimally coupled to gravity, these two observations pin down the Planck scale as the critical value. For the inflationary models with non-minimal (coupling to) gravity one has to determined the corresponding critical value in a given metric frame and then check how it changes from frame to frame.

We perform this study for the Higgs-inflation. We begin with the Jordan frame (JF), where settling the unitary gauge with Higgs boson h the model is described by the Lagrangian

$$S^{\text{JF}} = \int \sqrt{-g^{\text{JF}}} d^4x \left[-\frac{M_{\text{Pl}}^2}{16\pi} \left(1 + \frac{8\pi\xi h^2}{M_{\text{Pl}}^2} \right) R^{\text{JF}} + \frac{1}{2} g_{\mu\nu}^{\text{JF}} \partial^\mu h \partial^\nu h - \frac{\lambda}{4} h^4 \right]. \quad (1)$$

The observed matter perturbations fix the parameter of the non-minimal coupling to gravity at the value about $\xi \sim 5 \times 10^4$ [10] when the model is considered at the tree-level¹. A homogeneous (inside a given cell) configuration of the scalar field h changes the effective

¹We ignore here subtleties related to non-renormalizability of the couplings to gravity, treatment of the higher order corrections and behavior of the quantum effective potential at inflationary scale, referring to paper [13] for a comprehensive review.

Planck mass

$$M_{\text{Pl}} \rightarrow M_{\text{Pl}} \sqrt{1 + \frac{8\pi\xi h^2}{M_{\text{Pl}}^2}}$$

pushing with large value of h the scale of strong gravity up to higher energies,

$$E_{\text{GR}} \rightarrow \Lambda_{\text{JF}} \equiv \sqrt{8\pi\xi} h. \quad (2)$$

Hence, the critical value of energy density, E_{JF}^4 , that is the highest value consistent with the semiclassical treatment of the whole system, must not exceed Λ_{JF}^4 . The quantum uncertainty relation then allows for considering any cell of spatial size $l \geq 2\pi/E_{\text{JF}}$.

According to the chaotic inflation paradigm [5] all the ingredients contribute the same amount to the total energy density. At large values of h the proportional to ξ term in eq. (1) defines the gravity, the scalar kinetic and the scalar gradient contributions to the total energy density [12]. Consequently, the chaotic inflation implies the following initial conditions

$$\xi h^2 R^{\text{JF}} \sim \xi \dot{h}^2 \sim \xi (\partial_i h)^2 \sim h^4 \sim E_{\text{JF}}^4.$$

Thus we arrive at the estimate for the curvature $R^{\text{JF}} \sim h^2/\xi$ and with (2) and $\xi \gg 1$ one confirms that indeed the inflationary scale E_{JF} is below Λ_{JF} at large enough h , so the dynamics develops well inside the weak-coupling domain and the semi-classical treatment is fully applicable. *In the scalar sector (the Higgs boson and all the Standard Model fields) the strong coupling scale is lower than that in the gravity sector, and actually coincides with $E_{\text{JF}} \sim h$ [15, 14], which indeed defines the critical value, that is minimum of the strong coupling scales in all the sectors of the model, see Ref. [16] for a brief summary.*

Performing the Weyl transformation,

$$g_{\mu\nu}^{\text{JF}} \rightarrow g_{\mu\nu}^{\text{EF}} = \Omega^2 g_{\mu\nu}^{\text{JF}}, \quad \text{with} \quad \Omega^2 \equiv 1 + \frac{\Lambda_{\text{JF}}^2}{M_{\text{Pl}}^2}, \quad (3)$$

we come to the Einstein frame, where pure gravity term is Einsteinian with gravity scale M_{Pl} . It is shown in Ref. [12] that in this frame all the terms give equal contributions to the total energy density of order

$$E_{\text{EF}} \sim \Omega^{-1} \cdot E_{\text{JF}} \sim E_{\text{JF}} \cdot \frac{M_{\text{Pl}}}{\Lambda_{\text{JF}}} \sim \frac{M_{\text{Pl}}}{\sqrt{\xi}}.$$

Note, that the estimate of the critical value E_{EF} does not depend on the value of E_{JF} , while the dynamics is well inside the weak coupling domain. Recall again, the value of E_{EF} , that is

parameter ξ , is fixed from the cosmological observations. The energy density of the plateau-like scalar potential at inflation is $V_0 \sim M_{\text{Pl}}^4/\xi^2$ [10]. The minimum of the strong coupling scales in the Einstein frame coincides at inflationary stage with $E_{\text{EF}} \sim M_{\text{Pl}}/\sqrt{\xi}$ [15, 14], so it is indeed the critical value. The quantum uncertainty relation $\Delta E \Delta x \geq 2\pi$ implies that in the Einstein frame only a patch of the spatial size exceeding

$$1/E_{\text{EF}} \sim \sqrt{\xi}/M_{\text{Pl}} \gg 1/M_{\text{Pl}}$$

can be treated within semi-classical approach, providing the initial conditions for the classical equations describing evolution of the inflaton field and the Universe expansion. One concludes, that the initial conditions in the both Jordan and Einstein frames are truly in accord with the chaotic inflation paradigm.

4. Similarly one can address the issue of initial conditions in R^2 -inflation [1]. The model is described in the Jordan frame by the following Lagrangian

$$S^{\text{JF}} = -\frac{M_{\text{Pl}}^2}{16\pi} \int \sqrt{-g^{\text{JF}}} d^4x R^{\text{JF}} \left(1 - \frac{R^{\text{JF}}}{6\mu^2} \right). \quad (4)$$

The value of mass parameter μ is fixed from cosmological observations as $\mu \simeq 2.5 \times 10^{-6} \times M_{\text{Pl}}$. Similarly to the previous model an average curvature in a given cell changes the effective (local in a sense) Planck mass. The scale of strong gravity regime E_{GR} inside the cell also depends on the curvature R^{JF} . For the large enough values of the latter we have

$$E_{\text{GR}} \rightarrow \Lambda_{\text{JF}} \equiv \frac{M_{\text{Pl}}}{\sqrt{6}\mu} \sqrt{R^{\text{JF}}}. \quad (5)$$

For sufficiently large R^{JF} the first term in the brackets in Eq. (4) is negligible. Consequently, the contribution of the curvature generated by initial fluctuation of energy E_{JF} is estimated as

$$E_{\text{JF}}^4 \sim M_{\text{Pl}}^2 \frac{(R^{\text{JF}})^2}{\mu^2},$$

which implies the energy scale well below the strong gravity scale (5).

Now let us look at the theory in the Einstein frame, to which one can come by the same Weyl transformation (3) with Λ_{JF} given by Eq. (5). At this frame the energy of the initial fluctuation is

$$E_{\text{EF}} \sim \Omega^{-1} \cdot E_{\text{JF}} \sim E_{\text{JF}} \cdot \frac{M_{\text{Pl}}}{\Lambda_{\text{JF}}} \sim \sqrt{\mu} M_{\text{Pl}}.$$

As in the Higgs-inflation model it does not depend on the Jordan frame value while its energy density coincides with the value of plateau-like inflaton potential during inflation. Likewise,

the quantum uncertainty relation $\Delta E \Delta x \geq 2\pi$ implies that in the Einstein frame only a patch of the spatial size exceeding $l \sim 1/\sqrt{\mu M_{\text{Pl}}} \gg 1/M_{\text{Pl}}$ can be treated semi-classically.

5. To summarize, we have analyzed the frame-dependence of the initial conditions in inflationary models with modified gravity, emphasizing the importance of being in the weak coupling regime at the onset of expansion. With examples of the Higgs-inflation and R^2 -inflation we illustrated that the Weyl transformation between the frames properly changes the conditions (initial energy density, which is the same for all the terms), but keeps the model safely within the weak coupling domain. Naturally, the initial energy density, being frame-dependent, must not coincide with the Planck scale M_{Pl} , but rather be limited by the corresponding strong coupling scale. This conclusion seems to be general for the $F(R)$ modified gravity models and models with scalars non-minimally coupled to gravity, though an analysis similar to what is done above may be complicated by presence of several dynamical scalar fields.

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